Sets, Permutations and Combinations

Note Title 20/10/2006

Conventionally, sets are denoted using curly brackets:

$\{$Monday, Tuesday, Wednesday, Thursday, Friday$\}$

$\{0\}$

$\{$ red, yellow, blue $\}$

The Membership Relation

$x \in S$ means x is a *member* of (or *element* of) set S.

$\in$ denotes the *membership relation*.

The number of elements in a set S is denoted by $|S|$.

{Monday, Tuesday, Wednesday, Thursday, Friday}

$|\{0\}| = 1$

$|\{\text{red, yellow, blue}\}| = 3$

Cartesian Product

The cartesian product of sets S and T is denoted $S \times T$.

The elements of $S \times T$ are ordered pairs (s,t) where $s \in S$ and $t \in T$.

E.g. $\{a,b\} \times \{0,1\}$

$= \{(a,0), (a,1), (b,0), (b,1)\}$

Disjoint Sum

The disjoint sum of sets A and B is a set denoted by A+B. The elements of A+B are

- elements of A tagged with inl
- elements of B tagged with inr

Permutations (Ordering, Ranking)

A *permutation* of a set is a way of arranging the elements of the set in order.

Eg. {a,b} has two permutations: a b and b a.

{a,b,c} has six permutations:

a b c	b a c	c a b
a c b	b c a	c b a

S! set of permutations of S.

$|S!| = |S|! = |S| \times (|S|-1) \times (|S|-2) \times \ldots \times 1$

Subsets

Set S is a subset of set T, written $S \subseteq T$, if every element of S is an element of T.

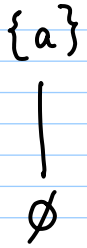
E.g.

$\{a\} \subseteq \{a, b\}$

$\{red, green\} \subseteq \{red, yellow, green, blue\}$

$\emptyset$ denotes the empty set.

$[\emptyset \subseteq S]$



Hasse diagram of $2^{\{a\}}$

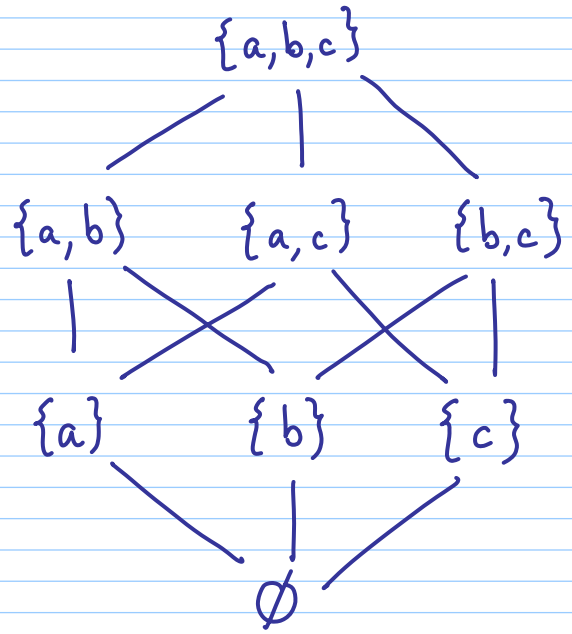
$2^{\{a\}} = \{\emptyset, \{a\}\}$

Hasse diagram of $2^{\emptyset}$

$2^{\emptyset} = \{\emptyset\}$

$\{\}$ is another notation for $\emptyset$.

Hasse Diagram of Subset Relation on $2^{\{a,b,c\}}$



Combinations

nCr is the number of subsets of cardinality r of a set of cardinality n.

— the number of ways of choosing r elements from a set with n elements.

$\binom{n}{r} = {}^nC_r$.

6 people are at the left bank of a river.
They want to get 3 of them across to the right bank.
Their boat will only carry 2 of them at a time.
How many different ways can this be done?